PG ENTRANCE EXAMINATION-2024(MATHEMATICS)

UNIT-1

1. **Successive Differentiation and Integral Calculus-I**: nth derivative of the standard function $e^{ax+b}, a^x, (ax+b)^n, \log(ax+b), \cos(ax+b), e^{ax}\sin(bx+c), e^{ax}\cos(bx+c)$, Leibnitz theorem and its applications. Recapitulation of definite integrals and its properties. Reduction formulae for $\int \sin^n dx, \int \cos^n dx, \int \sin^n x\cos^m xdx, \int \tan^n xdx$, $\int \cot^n xdx, \int \sec^n xdx, \int cosec^n xdx, \int x^n \sin xdx, \int x^n \cos xdx, \int x^n e^{ax}dx, \int x^n (logx)^m dx$ with definite limits.

2. **Integral Calculus-II:Line integral**: Definition of line integral and basic properties, examples on evaluation of line integrals. Double integral: Definition of Double integrals and its conversion to iterated integrals. Evaluation of double integrals by changing the order of integration and change of variables. Computation of plane surface areas using double integrals. Triple integral: Definition of triple integrals and evaluation- change of variables, volume as triple integral.

UNIT -2

1. **Number Theory:** Division Algorithm, Divisibility, Prime and composite numbers, Euclidean algorithm, Fundamental theorem of Arithmetic, The greatest common divisor and least common multiple. Congruences, Linear congruences, Simultaneous congruences, Euler's Phi-function, Wilson's, Euler's and Fermat's Theorems and their applications.

2. **Theory of equations**: Euclid's algorithm, Polynomials with integral coefficients, Remainder theorem, Factor theorem, Fundamental theorem of algebra(statement only), Irrational and complex roots occurring in conjugate pairs, Relation between roots and coefficients of a polynomial equation, Symmetric functions, Transformation, Reciprocal equations, Descartes' rule of signs, Multiple roots, Solving cubic equations by Cardon's method, Solving quartic equations by Descarte's Method.

3. **Polar Co-ordinates**: Polar coordinates, angle between the radius vector and tangent. Angle of intersection of two curves (polar forms), length of perpendicular from pole to the tangent, pedal equations. Derivative of an arc in Cartesian, parametric and polar forms, curvature of plane curve-radius of curvature formula in Cartesian, parametric and polar and pedal forms- center of curvature, circle of curvature.

UNIT-3

1. **Matrix**: Recapitulation of Symmetric and Skew Symmetric matrices, Algebra of Matrices; Row and column reduction to Echelon form. Rank of a matrix; Inverse of a matrix by elementary operations; Solution of system of linear equations; Criteria for existence of non-trivial solutions of homogeneous system of linear equations. Solution of non-homogeneous system of linear equations. Cayley- Hamilton theorem, inverse of matrices by Cayley-Hamilton theorem (Without Proof).

2. **Basics of Graph theory**: Basic definitions, Isomorphism, Subgraphs, Operations on graphs, Walks, Paths, Circuits, Connected and disconnected graphs, Euler graphs, Hamiltonian graphs, Some applications, Trees - basic properties, Distance, Eccentricity, center, Spanning trees, Minimal spanning tree.

UNIT-4

1. **Differential Calculus-I**: Limits, Continuity, Differentiability and properties. Properties of continuous functions. Intermediate value theorem, Rolle's Theorem, Lagrange's Mean Value theorem, Cauchy's Mean value theorem and examples. Taylor's theorem, Maclaurin's series, Indeterminate forms and evaluation of limits using L'Hospital rule.

2. **Partial Derivatives:** Functions of two or more variables-explicit and implicit functions, partial derivatives. Homogeneous functions- Euler's theorem and extension of Euler's theorem, total derivatives, differentiation of implicit and composite functions, Jacobians and standard properties and illustrative examples. Taylor's and Maclaurin's series for functions of two variables, Maxima-Minima of functions of two variables.

UNIT-5

1. **Differential Equations – I**: Recapitulation of Definition, examples of differential equations, Formation of differential equations by elimination of arbitrary constants, Differential equations of first order – Separation of variables, Reducible to separation of variables, Homogeneous differential equations, Reducible to homogeneous, Exact differential equations, Reducible to exact, Integrating factors found by inspection and the determination of integrating factors, Linear differential equations, Bernoulli's differential equations.

2. **Differential Equations – II**: Equations of First order and higher degree – Solvable for p, Solvable for x, Solvable y, Clairaut's equations – Singular and General solutions. Ordinary Linear differential equations with constant coefficients – Complementary function – particular integral – Inverse differential operators. Simultaneous differential equations (two variables with constant coefficients).

3. **Linear differential equations**: Cauchy – Euler differential equations, Solution of ordinary second order linear differential equations with variable coefficients by various methods such as: (i) When a part of complementary function is given. (i) Changing the independent variable. (ii) Changing the dependent variable. (iii) By method of variation of parameters. (iv) Exact method. Total differential equations - Necessary and sufficient condition for the equation Pdx + Qdy + Rdz = 0 to be exact (proof only for the necessary part) – Simultaneous equations of the form dx/ P = dy/ Q = dz/ R.

4. **Partial differential equations**: Basic concepts – Formation of a partial differential equations by elimination of arbitrary constants and functions – Solution of partial differential equations – Solution by Direct integration, Lagrange's linear equations of the form Pp + Qq = R , Standard types of first order non-linear partial differential equations – Charpit's method – Homogenous linear equations with constant coefficient – Rules for finding the complementary function – Rules for finding the particular integral, Method of separation of variables (product method).

UNIT-6

1. **Group Theory – I** :Definition and examples of groups – Some general properties of Groups, Subgroups, Group of permutations – Cyclic permutations – Even and odd permutations. Order of an element of a group – Cyclic groups problems and theorems.

2. **Group Theory – II** : Cosets, Index of a group, Lagrange's theorem, consequences, Normal Subgroups, Quotient groups – Homomorphism. – Kernel of homomorphism – Isomorphism - Automorphism – Fundamental theorem of homomorphism, Cayley's theorem.

3. **Rings and Fields Rings** – definition and properties of rings- integral domains- Fields-theorems and problems, Sub rings- Criterion for sub rings- theorems and problems on sub rings, Ideals –Algebra of Ideals-theorems-Principal ideals - examples and standard properties following the definition, Divisibility in an integral domain-theorems and problems, Units and Associates- theorems and problems. Quotient rings– examples and theorems-The field of quotients-theorems and problems.

4. **Homomorphism of a ring Homomorphism-** Definitions and example, Kernel of a homomorphism- examples and related theorems. Isomorphism of a ring- examples and related theorems. Automorphism- problems. Fundamental Theorem of Homomorphism of Rings, Prime and Maximal ideals in a commutative ring - examples and standard properties following the definition. Unique factorization domain-examples and theorems.

5. **Polynomial rings**: Polynomials over rings and fields (some standard properties), division algorithm (proof and problems), Greatest common divisor – Euclidian algorithm (problems); reducible and irreducible polynomials over fields (definition and problems); Eisenstein's criteria for reducibility – Proof and problems; Rational roots of a polynomial – Test (proof and problems); Unique factorization theorem– Proof.

UNIT-7

1. **Complex number** – Cartesian and Polar form (Definitions, properties and problems)- Geometrical representation of complex plane (z-plane); Euler's formula, $e^{i\theta} = \cos\theta + i\sin\theta$, Separate the real and imaginary parts of some standard functions($e^z, \sin z, \cos z, \log z$ etc)Dot and vector product of z_1 and z_2 . Equation of a straight line and circle in a complex form andrepresent graphically (locus of a point). Functions

of a complex variable - Limit of a function, Continuity and differentiability, Analytic functions, Singular points (definitions and related problems) Cauchy-Riemann equations – Cartesian and Polarforms – Proof and Problems, Necessary and sufficient condition for a function to be analytic (Statementonly); Harmonic functions– Definition and problems; Properties of analytic functions - Various properties with proofs; Construction of analytic functions: i)MilneThomson Method (Only problems) ii)Using the concept of harmonic function.

2. **Complexintegration**: Complex integration– definition, Line integral, properties and problems. Cauchy's Integral theorem- proof using Green's theorem- direct consequences. Cauchy's Integral formula with proof-Cauchy's generalized formula for the derivatives with proof and applications for evaluation of simple line integrals. Cauchy's inequality- Proof, Livouville's theorem- Proof.

3. **Transformations**: Definition, Jacobian of a transformation- Identity transformation- Reflection- Translation-Rotation and Magnification- Inversion- Inverse points- Linear transformation- Definitions- Bilinear transformations-Cross- ratio of four points- Cross-ratio preserving property- Preservation of the family of straight lines and circles-Conformal mappings- Discussion of the transformations $w = z^2, w = \sin z, w = \cos z, w = e^z, w = \frac{z+\bar{z}}{2}$ etc

UNIT-8

1. **Vector spaces**: Vector spaces - Definition, examples and properties; Subspaces - Examples, criterionfor a subspace and some properties; Linear Combination - Linear span, Linear dependence and Linear independence, basic properties of linear dependence and independence, techniques of determining linear dependence and independence in various vector spaces and related problems; Basis and dimension - Co-ordinates, ordered basis, some basic properties of basis and dimension and subspaces panned by given set of vectors; Quotient space- theorems and examples.

2. **LinearTransformations**: Linear transformation - Definition, examples, equivalent criteria, some basic properties and matrix representation, change of basis and effect on associated matrix, similar matrices; Rank - Nullity theorem -Null space, Range space, proof of rank nullity theorem and related problems.

3. **Isomorphism, Eigen values and Diagonalization** Homomorphism, Isomorphism and automorphism-Examples, order of automorphism and Fundamental theorem of homomorphism; Eigen values and Eigen vectors-Computation of Eigen values, algebraic multiplicity, some basic properties of eigen values, determination of eigen vectors and eigen space and geometric multiplicity. Diagonalizability of linear transformation - Meaning, condition based on algebraic and geometric multiplicity (mentioning) and related problems (Only verification of diagonalizability).

4. **InvertibleTransformation and Inner product spaces Invertible transformation** - some basic properties of Invertible, singular and non-singular transformations and conditions for existence of inverses; Minimal polynomial of a transformation. Relation between characteristic and minimal polynomials and related problems. Inner product and normed linear spaces- Definitions, examples, Cauchy-Schwartz

   inequality (withproof) and related problems; Gram-Schmidt orthogonalization- Orthogonalvectors, orthonormal basis, Gram-Schmidt orthogonalization process: both proof

   and problems.

UNIT-9

1. **Algebraic and Transcendental Equations Errors**- Significant digits, absolute, relative, percentage errors, rounding off and truncation errors (meanings and related problems), general error formula (derivation of formula and problems based on it), error in series approximation: Taylor series approximations (problems only), Solutions to algebraic and transcendental equations - Bisection method, Regula-Falsi method, iterative method Newton-Raphson method and secant method (Plain discussion of the rationale behind techniques and problems on their applications).

2. **System of LinearAlgebraicEquations Direct Methods**– Gauss elimination method, Gauss-Jordan elimination method and Tringularization method; Iterative methods – Jacobi method, Gauss-Jacobi method, Gauss-Seidal method,Successive- Over Relaxation method(SOR) method.

3. **Polynomial Interpolations**: Finite differences. Forward, backward and central differences and shift operators: definitions, properties and problems; Polynomial interpolation - Newton-Gregory forward and backward interpolation formulas, Gauss's Forward and backward interpolation formulas, Lagrange interpolation polynomial, Newton's divided differences and Newton's general interpolation formula (Discussion on setting up the polynomials, differences between them and problems on their applications).

4. **Numerical Differentiation and Integration**: Formula for derivatives (till second order) based on Newton-Gregory forward and backward interpolations (Derivations and problems based on them). Numerical Integration-General quadrature formula, Trapezoidal rule, Simpson's1/3rule, Simpson's 3/8 rule and Weddell's rule (derivations for only general quadrature formula, trapezoidal rule and Simpson's 1/3rd rule and problems on the applications of all formulas).

UNIT -10

1. **Sequences** : Sequence of real numbers – Bounded and unbounded sequences – Infimum and supremum of a sequence – Limit of a sequence – Sum, product and quotient of limits – Standard theorems on limits – Convergent , divergent and oscillatory sequences – Discuss the convergence of $x^n, n^{\frac{1}{n}}, \left(1+\frac{1}{n}\right)^n$ and standard problems – Monotonic sequences and their properties – Cauchy's general principle of convergence.

2. **Infinite Series**: Infinite series of real numbers – Convergence and Divergence - Oscillation of series –Properties of convergence – Series of positive terms – Geometric series – p – series – Comparison test– D'Alembert's ratio test – Raabe's test – Cauchy's root test – Leibnitz's test for alternating series.

3. **RiemannIntegration-I**: Definition and examples for partition of an interval, Refinement and Common refinement of a partition. Lower and Upper Riemann (Darboux) sums – definition, properties and problems. Riemann Integral– Lower and Upper integrals (definition and problems), Darboux's theorem and Criterion for Integrability, Integrability of sum, difference, product, quotient and modulus of integrable functions. Integral as a limit of sum (Riemannsum) –Problems. Some integrable functions –Integrability of continuous functions, monotonic functions, bounded function with finite number of discontinuity. Fundamental theorem of Calculus–related problems, change of variables, integration by parts, first and second mean value theorems of integral calculus.